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This final report summarizes the findings of research on the topic "Stochastic Dynamic Systems with Multiple Decision Makers and Parametric Uncertainties," supported by a Grant from the Air Force Office of Scientific Research, during the period May 1, 1985-April 30, 1988. The focus of the research during this three-year period has been on the development of methodologies and new solution techniques for obtaining strategies in stochastic systems, with good sensitivity properties, and for deriving optimal decision rules in systems with nonclassical information patterns. A further major thrust has been on the development of learning schemes and distributed algorithms for multiple decision-maker problems under different types of uncertainty. (kr)

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STOCHASTIC SYSTEMS WITH MULTIPLE DECISION MAKERS AND PARAMETRIC UNCERTAINTIES

FINAL REPORT TO AFOSR
covering the period
May 1, 1985-April 30, 1988

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ABSTRACT

This final report summarizes the findings of research on the topic "Stochastic Dynamic Systems with Multiple Decision Makers and Parametric Uncertainties", supported by a Grant from the Air Force Office of Scientific Research, during the period May 1, 1985 - April 30, 1988. The focus of the research during this three-year period has been on the development of methodologies and new solution techniques for obtaining strategies in stochastic systems, with good sensitivity properties, and for deriving optimal decision rules in systems with nonclassical information patterns. A further major thrust has been on the development of learning schemes and distributed algorithms for multiple decision-maker problems under different types of uncertainty.

1. Multiple Decision-Maker Problems with Unknown Parameters

The problem of strategic decision making in complex systems which involve multiple decision makers (DM's), multiple objectives, and incomplete information arises frequently in the military context. As compared with single DM problems, the analysis of multiple DM problems requires different approaches and techniques, and furthermore certain standard features and properties we usually ascribe to single DM problems do not generally extend naturally to multiple decision making. For example, while, in single DM problems, optimization (minimization or maximization) of a single objective functional would, in general, lead to a satisfactory decision policy (the so-called *optimal policy*), when the decision problem involves multiple DM's and multiple objectives a plethora of possibilities emerge as to the criterion which leads to a "satisfying" set of policies. Depending on the number of DM's, their underlying goals, and the presence or absence of dominance in the decision making process, we may have *team-optimal*, *person-by-person optimal*, *Pareto optimal*, *Nash equilibrium*, *Stackelberg* (leader-follower) *equilibrium*, *consistent conjectural variations equilibrium* concepts, and several variants of combinations of these in case of more than two DM's. Each of these leads, in general, to a different outcome which is also a variant of the information structure of the problem (i.e., what each DM knows *a priori*, what information he acquires during the evolution of the decision process, what information exchange links are allowable, and what information transmission capability each DM is vested with). The significance of information structure in multiple DM problems also manifests itself in the derivation of multimodel strategies: Model simplification through singular perturbations or aggregation is not a well-posed procedure unless there is some kind of a matching between the information structures of the original problem and the simplified version; no such inconsistencies arise, however, in single decision-maker problems.

Recent years have witnessed considerable advances in our understanding of equilibrium solutions of deterministic and stochastic multiperson decision problems, and in particular as regards the Stackelberg equilibrium solution. A class of such Stackelberg problems which were long thought to be extremely challenging have been solved using indirect methods, for both deterministic and stochastic systems. In some cases it has been shown that the Stackelberg equilibrium strategy for the leader forces the DM's at lower levels of hierarchy to a team behavior, jointly optimizing the leader's performance index, even though they may each have different goals and performance indices. In other cases, tight performance bounds have been obtained on the leader's cost function, which are achievable by implementable policies.

A large majority of this work on multiple DM problems pertains to either deterministic systems or to systems with uncertain elements which have a complete probabilistic description—this *a priori* information being known by all the DM's (the latter class of problems are also known as stochastic dynamic games). Hence, even though some decentralization of dynamically acquired information has been allowed for in the general formulation of dynamic games, it has been a common assumption to endow every DM with the common (centralized) *a priori* information regarding the complete statistical description of the "primitive" random variables. Our thesis has been that such an underlying assumption is not always a realistic one, especially when the decision problem involves distributed tasks for the DM's. A more realistic formulation, in most cases, would allow for discrepancies in the perceptions of the DM's regarding the underlying stochastic model. These discrepancies could be accommodated in the model by having a number of parameters which are either not stochastic or are stochastic but their complete statistical description is not known by all the DM's.

The presence of unknown (or uncertain) parameters could affect the general problem formulation in basically three different ways:

i) *Through the objective functions.* Here, the objective function of the i'th DM may not be known completely by the j'th DM ($j \neq i$), with the uncertainty characterized by a number of parameters whose values are unknown to the j'th DM.

ii) *Through the system response.* The evolution of the decision process may depend on a number of parameters whose values are unknown to some or all DM's. [This type of uncertainty is also applicable to stochastic team problems.]

iii) *Through the measurements made by the DM's.* Here either the observation scheme or the statistics of some of the variables in the measurement process of a DM (or both) may not be known to some other DM, with the uncertainty again being parameterized. [As in ii) this type of uncertainty is also applicable to stochastic team problems.]

Multiple DM problems with the types of uncertainties as described above can be treated by adopting essentially one of the following three approaches:

a) *Robustness or Minimum Sensitivity Approach.* Here one assumes some nominal values for the unknown parameters, determines a corresponding nominal performance for the system, and designs decision policies which would lead to minimum performance degradation should the parameters vary around their nominal values. The resulting decision policies are called *minimum sensitivity strategies*, and they are robust in a certain neighborhood of the nominal values. For some recent advances in this area and for motivation of this approach we refer to publications [P1],[P2] and [P11], which report our recent work on this topic, carried out under AFOSR support.

b) *Learning Schemes.* In this approach no nominal values for the unknown parameters will be available, but some *a priori* statistics may be attached to these parameters by the DM's, which will be updated in a decentralized manner as new dynamic information is acquired. This is akin to some of the methodologies developed earlier for control problems with unknown parameters (such as identification, parameter estimation, and adaptive control--still active research areas), which are, however, not applicable to multiple DM problems, because the rather intricate interactions of multiple DM's render any central learning scheme infeasible. The iterative schemes which are needed for such systems have to be decentralized and distributed, and have also to account for the possibility that DM's may not update their policies or actions in a predetermined order. There are the further questions of robustness of these schemes to possible inaccuracies in the computation phase during each update, and robustness to environmental changes. Learning schemes could involve two types of iterations: "iteration in the policy space" and "iteration in the decision (action) space". During the past three years we have devoted considerable attention to the former, under AFOSR support; our accomplishments in this area are discussed in some detail in the next section. Furthermore, we have made considerable progress on the second type of iteration referred to above, as also elucidated in the next section.

c) *Minimax Approach.* Here no nominal values are available for the unknown parameters, but they are known to belong to some pre-specified sets. Then, the objective is to design strategies which would carry optimality or equilibrium property under worst possible values of the parameters on these sets. [See, for example, [P9] and [P10] for two different contexts where such a formulation would arise.] Such an approach entails a pessimistic design philosophy, and is applicable mostly to decision problems with a common objective functional (i.e., team problems). In multi-objective problems, the minimax philosophy may lead to some ambiguity, since what may seem to be a worst-case design for one objective functional may seem to lose this property when tested against a different objective functional. However, if different objective functionals are affected by different sets of unknown parameters, this approach would still be applicable. Furthermore, in some information transmission problems where the channel description is not complete, the minimax design (transmission) philosophy finds a natural home, as elucidated particularly in the recent paper [P23].

We should point out that a combination of any two or all three of the above approaches would also constitute a viable approach to multi-person decision problems with unknown parameters, which should be studied in proper contexts once the rudiments of a theory for each one separately is laid down.

2. Research Accomplishments

In our proposals for this research, carried out under AFOSR support during the past three years, we had recognized the fact that the class of multiple DM problems with uncertain parameters, as described above, are still in their infancy, in particular under the "Learning Scheme" approach, and when the underlying information patterns are nonclassical. In view of this, we proposed to conduct original fundamental research to make theoretical advances in this field, of both methodological and algorithmic nature, and to design implementable decision policies which carry both the learning and command capabilities. To accomplish this, we proposed to adopt the general framework of deterministic and stochastic dynamic games, and to conduct a study under the three types of uncertainty introduced in Section 1, using different solution concepts such as team-optimal, Nash equilibrium and Leader-Follower (hierarchical). We placed emphasis on the development of decentralized and distributed schemes with learning capabilities, and also proposed to study the issues of optimal and suboptimal strategy design in stochastic control and team problems of the nonstandard type, especially those with nonclassical information patterns, so as to enhance our understanding of the intricate role played by information patterns, active and passive learning, and hierarchies in such problems.

During the past three years, we have addressed several challenging issues in this context, and have made important strides. We provide below a brief summary of our research findings, full details of which can be found in the references listed in Section 3. Copies of all the references with publication dates of May 1987 or later are attached (in full) to this report; for publications of the two previous years, we attach to this report only a selected number of them (those with asterisks), since they were all submitted earlier along with the two previous progress reports.

We now return to brief descriptions of the main contributions of the papers listed in Section 3. In the first group of papers, listed in Section 3 as [P1]-[P3], we have adopted the first (i.e., minimum sensitivity) approach for a class of decision problems which displayed the first type of uncertainty, viz. the case of one of the DMs' cost function depending on a number of parameters whose precise values are known by him but not by other(s). In [P1], we have presented a general mathematical formulation and a method of solution for stochastic incentive decision problems, using concepts and tools of dynamic game theory. As special cases of the general formulation we have considered four different classes of problems which differ in the information available to the DM's, their objectives, and the number of DM's at different levels of hierarchy. The fourth class we considered can be viewed as an "exact model matching" problem akin to the one arising in nonlinear control. In the paper, an explicit incentive policy has been obtained for the DM occupying the higher level in the hierarchy, which, besides solving the exact matching problem, carried very appealing minimum sensitivity properties. These features have also been demonstrated in [P1] in the context of a numerical example. In [P2] we have extended these results to more general models, involving decision problems defined on finite dimensional spaces. In [P3] we have considered stochastic decision problems with two levels of hierarchy, and $N>1$ DM's (followers) occupying the lower level. We showed that if the leader's dynamic information comprises only a linear combination of the followers' actions, he can design a policy, affine in this dynamic information, which yields the same overall performance as the one the leader would obtain had he observed the followers' actions separately. This is a feature intrinsic to stochastic problems and have no counterpart in deterministic systems. In the paper we have presented explicit solutions and existence conditions for the case of finite probability spaces, and have identified several challenging issues when the random variables are defined on infinite spaces. In more recent work, reported in [P11], we extended the results of [P2] to multi-stage decision processes where the objective functionals of the DM's depend on a time-varying uncertain parameter. We have obtained strategies that use the past values of the state measurements (i.e. memory), which desensitize the performance against variations of the uncertain quantities about their nominal values.

The next two papers, [P4] and [P5], address a different class of problems, where the uncertainty is of the second and third types (see Section 1), and the general approach is the "learning scheme"; here, three solution concepts, viz. Nash, hierarchical, and Pareto-optimal, are employed. While the discussion in [P4] pertains to two-person decision problems, the sequel [P5] is devoted to the general N-agent case. The analyses cover both finite and infinite-state models, where the uncertainties are in the statistical description of the random variables appearing in the system dynamics and the measurements of the two DM's, and the DM's are allowed to develop different prior probabilities on these random variables. The papers develop different recursive schemes which involve "learning in the policy space", and lead to policies that converge to the equilibrium under different stipulations on the information structure of the problem. We have also analyzed the robustness and sensitivity of team optimal solutions to deviations in the perceptions of the DM's from a common stochastic model, and have shown that adoption of the Nash equilibrium solution leads to well-posed models, whereas the other two solution concepts lead to bifurcation once deviated from the nominal model. An important by-product is a convergent algorithm which yields the optimal solution of a quadratic stochastic team problem with decentralized information, in which the underlying statistics are not Gaussian. Counterparts of these results in the case of n-person continuous-time stochastic differential games have been obtained more recently in [P12] where, as before, we have allowed for uncertainties in the statistical description of the random variables appearing in the system dynamics and the measurements of the n decision makers (players). Furthermore, the players were allowed to develop multiple (possibly inconsistent) probabilistic models for the underlying system (state dynamics and measurement equations). We obtained conditions for the existence and uniqueness of Nash equilibrium, and developed a method for iterative distributed computation of the solution. The distributed algorithm presented in [P12] involves learning in the *policy space*, and it does not require that each player know the others' perceptions of the probabilistic model underlying the decision process. For the finite horizon problem, such an iteration converges whenever the length of the time horizon is small, and the limit in this case is an affine policy for all players, if the underlying distributions are Gaussian. When the horizon is infinite, and a discount factor is used in the cost functionals, the iteration converges under conditions depending on the magnitude of the discount factor, the limiting policies again being affine in the case of Gaussian distributions.

Papers [P8] and [P13] deal with the development of distributed computational schemes for <u>nonlinear</u> <u>nonquadratic</u> multi-person decision problems. One of the important results is the derivation of a general condition (called *persistent contraction*) under which a number of iteration schemes in two DM problems converge to the desired equilibrium, when the cost functionals are nonquadratic and the DM's do not necessarily adopt the same model. The algorithms developed use both accurate and inaccurate search techniques in the policy space, and apply to discrete-time as well as continuous-time decision problems. A number of numerical examples included in the two papers, as well as in the thesis [T1], illustrate different features of these algorithms, and in particular their superiority over both Newton and gradient type algorithms.

The iterative schemes studied in [P8] and [P13] all lead to an equilibrium solution provided that it is *stable*. Roughly speaking, we say that an equilibrium solution of a zero-sum or a nonzero-sum game is *stable* if, after any deviation from that equilibrium, an adjustment process that involves unilateral optimal responses by the players can bring it back to the starting point. One appealing feature of a stable equilibrium is that in the on-line adjustment process each DM (or player) need to know only his own cost function and the most recently computed (and broadcast) policies of the other players, and not the other players' cost functions. It is needless to say that not all (saddle-point or Nash) solutions are stable, and hence the question arises as to whether there exists a different on-line (real-time implementable) computational algorithm, than those discussed in [P8] and [P13], which would converge to an equilibrium even if that equilibrium is not stable. In [P14] we have addressed precisely this question, and have introduced a relaxation technique which leads to on-line implementable algorithms that converge to equilibria, be they stable or not, and in some cases in a finite number of steps. In the paper, we have also obtained conditions for the convergence of asynchronous algorithms, which arise in the computation of equilibria in games where the order of responses is not fixed *a priori*. The discussion and analyses are confined primarily to two-person deterministic problems, with extensions to n-person games and stochastic games identified as challenging problems for future research.

The next paper in the list, [P15], addresses the second type of iteration and learning scheme mentioned in Section 1: "iteration in the decision (action) space". Here our model is a network of processors (DM's) connected by partial communication links and engaged in distributed computation. They receive information, make decisions based on the information in their buffer, and transmit the decision to a subset of other DM's, all at random points in time. In the paper, we present results on the convergence and asymptotic agreement of a general class of asynchronous algorithms which arise in this context. These algorithms are in general time-varying, memory-dependent, and not necessarily associated with the optimization of a common cost functional. We obtain the precise conditions under which convergence to a unique set of decisions and asymptotic agreement on the shared information can be reached by distributed learning. It is shown that a separation of fast and slow parts of the algorithm is possible, leading to a separation of the estimation process from the computation phase. These results are obtained using random contraction mapping arguments and under various meaningful assumptions on the sizes of buffers for the DM's to store the information they receive. More precisely, convergence is established even if the memory of each processor is bounded, whereas asymptotic agreement is achieved when there are no limitations imposed on the memory and computational capabilities of the processors.

As we have mentioned earlier, one of the most challenging problems in multiple-person decision making is the problem of obtaining optimal solutions to stochastic teams with nonclassical information. In a proper context, these problems may also be referred to as "stochastic control problems with nonclassical information". One of the most important and mostly referenced works in stochastic control is the 1968 paper by Witsenhausen, where a counterexample was given to refute the common belief that all LQG (linear quadratic Gaussian) control problems admit linear solutions. For a scalar example with nonclassical information pattern, it was shown that the best linear solution can be outperformed by a nonlinear solution, and more importantly that the derivation of the best nonlinear solution (which exists) is a most challenging task, even numerically. Almost twenty years have passed since then, and the question of the best solution for that specific problem is still open. There has also remained the further important question of the extent of validity of the features displayed in that paper for the general class of stochastic control or team problems; in other words, are all stochastic team problems with nonclassical information patterns inherently difficult and complex? In [P16] we have shed some light on these questions, and have obtained some fundamental results. Specifically, we have considered a parameterized class of two agent team problems with strictly nonclassical dynamic information, which also includes the earlier formulation, and in this class we have identified a subclass which admits a linear solution. When we go outside this subclass, we show that the best linear solution can be outperformed by a linear combination of linear and piecewise constant control laws, and we re-interpret the basic result of Witsenhausen in this more general framework. Hence, we have a complete partitioning of the parameter space into two regions, in one of which the optimal solution is linear, and in the other it is inherently nonlinear, but a piecewise constant type control law alone does not always improve upon the best linear law. In proving the optimality of linear laws for the first class, we have adopted a unique approach and have used some results from information theory.

In [P17], we have extended the findings of [P16] in new important directions. Specifically, we consider in that paper a stochastic dynamic team problem with two controllers and nonclassical information, which can be transmitted as the transmission of a garbled version of a Gaussian message over a number of noisy channels, under a given fidelity criterion. We show that the optimal solution (under a quadratic loss functional) consists of linearly transforming the garbled message to a certain (optimal) power level, and then optimally decoding it by using a linear transformation at the receiving end. The optimum power level alluded to above is determined by the solution of a fifth-order algebraic equation. The paper also discusses an extension of this result to the case when the channel noise is correlated with the input random variable, and it shows that the solution is again linear for the single channel case.

These results on two stage stochastic teams with nonclassical information have subsequently been used in our research towards developing a general theory for multi-stage (finite and infinite horizon) stochastic control and team problems with nonclassical information, where the control (decision) variable does not only affect the state trajectory but also the quality of information available to the decision makers, thus exhibiting a dual role. Two papers which report research results in that direction are [P18] and [P19], which deal with two totally different classes of stochastic decision problems with nonclassical information, and obtain explicit solutions in both cases using two totally different approaches.

In [P18] we consider a stochastic dynamic decision problem where at each step two consecutive decisions must be taken, one being what information bearing signal to transmit, and the other what control action to exert. Such a problem arises in the simultaneous optimization of both the observation and the control sequences in stochastic systems, and is a prime example of a stochastic team problem with nonclassical information. Using bounds from information theory, we were able to solve this problem completely for first-order systems under a quadratic cost criterion. We have shown in [P18] that, in the case of hard power constraints, the optimal measurement policy consists of transmitting the "innovation" in the new data at the maximum power level. In the case when the power levels at the transmitter are not fixed, the optimal power levels for transmitting this innovation can be found by solving a nonlinear optimal control problem. These results are further extended in [P20] to the case when the time horizon is infinite and the cost functional is discounted, in which context we prove the existence of optimal stationary control and transmission strategies, and provide a complete characterization of the optimal solution. An extension in a different direction is provided in [P21], where we study stochastic teams with (i) more than one DM (agent) who performs the communication task of generating information bearing signals, and (ii) more than one agent performing the control function.

The problem treated in [P19] is another stochastic dynamic optimization problem which exhibits active learning, but the derivation of its solution requires a completely different approach. The proof of optimality given in the paper relates the original single objective problem to a sequence of nested zero-sum stochastic games. Existence of saddle points for these games implies the existence of optimal policies for the original stochastic control problem, which, in turn, can be obtained from the solution of a nonlinear deterministic optimal control problem. The paper also studies the problem of existence of stationary optimal policies when the time horizon is infinite and the objective function is discounted. This is one of the first reports in the literature on the derivation of closed-form solutions for a class of (non-neutral) stochastic control problems where the control directly affects the quality of information carried to future stages. In a subsequent work reported in [P22] we have provided a different perspective to the results of [P18] and [P19] by introducing a unifying framework and also analyzing the contrasts between the formulations and the method of solutions in the two papers.

In [P23], we expand on the framework of [P16] and [P17] by allowing incomplete statistical information on some of the variables and seeking an optimal solution under a worst case analysis. We again operate under nonclassical information patterns and consider a number of cases depending on whether there are "hard" energy constraints or "soft" constraints on some decision variables and/or "soft" costs on communications. We obtain minimax decision rules in all these cases, some being saddle points and others not, the techniques of derivation being very much case-dependent. Further results on this general class of problems have recently been reported in [P24], where the optimal (saddle-point) solution dictates the use of a probabilistic decision rule. These are all important prototype problems which could be considered essential building blocks for a general theory of multi-stage distributed decision making under nonclassical information, and with partial statistical description.

Two other papers in the list, which deal with the minimax philosophy are [P9] and [P10]. In [P9] we address the problem of designing time-invariant controllers for stochastic systems with parametric uncertainties, under both the minimax and minimum sensitivity approaches. We first introduce some applicable theory, and then develop some numerical algorithms to obtain both the minimax and minimum sensitive controllers, under different assumptions on the order of the controllers. A comparative analysis of the numerical results displays some interesting features of the solution, which are discussed at length in the paper. In [P10] we obtain a fundamental result for stochastic decision problems with unknown parameters under a worst case design philosophy. For a fairly general class of sequential decision processes, we show that even in the absence of a saddle point, the min-max strategy can be obtained by means of a dynamic programming type recursion. In addition to proving this theorem, we also examine the precise roles of the strategy sets allowed to the minimizer and the maximizer in determining the min-max value.

The sixth paper in the list, [P6], addresses a decentralized large scale decision (team) problem with N DM's, and introduces a novel procedure to obtain suboptimal policies with appealing features. It utilizes the method of chained aggregation to decompose the overall team problem into (N+1) subproblems: one low order team problem with a centralized information structure and N decentralized optimal control problems. Accordingly, the control of each DM is decomposed into three components: a decoupling control which induces aggregation, a local control which controls the subsystem dynamics, an aggregate control which controls the dynamics of the interconnection variables. The paper also establishes the robustness of this composite control with respect to perturbations in the system dynamics and the cost functional.

The seventh paper, [P7], is a tutorial, based on an opening talk given by the PI at a conference in London in June 1985, and it presents in a nutshell rudiments of the theory of multi-person decision making in a dynamic environment. The paper does not only survey the literature, but also introduces a mathematically rigorous definition for a new solution concept — *consistent conjectural variations equilibrium.* This solution opens up new challenging problems in this area, some of which are identified in the paper. In another tutorial paper, [C3], which is an opening plenary talk given at an IFAC Workshop in Beijing in August 1986, we provide a survey on the optimum design of organizations with decentralized information, and identify a number of challenging issues with regard to existence of smooth incentive schemes and their robustness.

Finally, the two recent papers [P26] and [P27] study and develop a new approach for policy optimization problems that involve the so-called "forward-looking" stochastic models. Such models provide a characterization of decision processes where the evolution of the underlying dynamics depends explicitly on the expectations the controlling agents form on the future evolution itself. They lead to nonstandard stochastic dynamic optimization problems where one has to take into account the fact that there is a circular (closed) relationship between future forecasts and the future system behavior. In [P26] we study the class of models where the only input involves a two-step ahead prediction of the future system behavior, by formulating them as stochastic control problems (of the delayed information type) in both finite and infinite horizons. It is shown that when there is perfect state information, the solution is unique for both the finite and infinite horizon formulations, and it requires memory for the former while requiring only current state information for the latter. When only noisy measurements are available, it is shown that a certainty-equivalence type result holds, the memory requirements being the same as above, with perfect state now replaced by the output of a finite-dimensional filter. The second paper, [P27], extends these results to forward-looking models which have two types of controlled inputs -- a forecasting strategy and a tracking strategy. The objective of the second control input is to make the system track a given trajectory. This leads to a game-theoretic formulation, which we thoroughly study in [P27] for both finite and infinite horizons, and under both perfect and noisy state measurements. In all cases we show that the problem admits a unique Nash equilibrium solution, and provide a complete characterization of the corresponding decision rules.

3. Publications Supported by the Grant

Papers

(Only the ones with asterisks are attached to this report.)

[P1] Tamer Basar, "Dynamic Games and Incentives," in *Systems and Optimization, Lecture Notes in Control and Information Sciences*, A. Bagchi and H. T. Jongen (eds.), Springer-Verlag, Vol. 66, 1985, pp. 1-13.

*[P2] Derya H. Cansever and Tamer Basar, "Optimum/Near-Optimum Incentive Policies for Stochastic Decision Problems in the Presence of Parametric Uncertainty," *Automatica*, Vol. 24, No. 5, September 1985.

*[P3] Derya H. Cansever and Tamer Başar, "On Stochastic Incentive Control Problems with Partial Dynamic Information," in *Systems and Control Letters*, Vol. 6, June 1985, pp. 69-75; also in *Proc. 24th IEEE Conference on Decision and Control*, December 1985, pp. 564-567.

[P4] Tamer Başar, "Stochastic Multimodeling for Teams in a Game-Theoretic Framework," in *Optimal Control Theory and Economic Analysis 2*, G. Feichtinger (ed.), 1985, pp. 529-548.

[P6] V.R. Saksena and J.B. Cruz, Jr., "An Approach to Decentralized Control of Large Scale Systems Using Aggregation Methods," *IEEE Transactions on Automatic Control*, Vol. Ac-30, No. 9, September 1985, pp. 912-914.

[P7] Tamer Başar, "A Tutorial on Dynamic and Differential Games," in *Lecture Notes in Economics and Mathematical Systems*, T. Başar (ed), Springer-Verlag, March 1986, pp. 1-25.

[P8] Shu Li and Tamer Başar, "Distributed Learning Schemes for Nash Games," *Proceedings, 1986 IFAC Workshop on Games and Decisions*, Beijing, China, August 1986; pp. 707-316.

[P9] Paul H. McDowell and T. Başar, "Robust Controller Design for Linear Stochastic Systems with Uncertain Parameters," *Proc. 1986 American Control Conference*, June 1986, pp. 39-44.

*[P10] Tamer Başar and P. R. Kumar, "On Worst Case Design Strategies," *Computers and Mathematics with Applications*, Vol. 13, No. 1-3, 1987, pp. 239-245.

*[P11] Derya H. Cansever and Tamer Basar, "Parametric Insensitivity of Equilibria in Multi-stage Dynamic Games," *Proceedings of the 10th IFAC World Congress*, Munich, W. Germany, July 1987, Vol. 5, pp. 215-220.

*[P12] Tamer Başar and Shu Li, "Distributed Computation of Nash Equilibria in Linear Stochastic Differential Games," *Proceedings of the 10th IFAC World Congress*, Munich, W. Germany, July 1987, Vol. 9, pp. 231-236; also currently under review for a journal publication.

*[P13] Shu Li and Tamer Başar, "Distributed Algorithms for the Computation of Noncooperative Equilibria," *Automatica*, Vol. 23, No. 4, July 1987, pp. 523-533.

*[P14] Tamer Başar, "Relaxation Techniques and Asynchronous Algorithms for On-Line Computation of Noncooperative Equilibria," *Journal of Economic Dynamics and Control*, Vol. 11, December 1987; pp. 1-19.

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*[P17] Rajesh Bansal and Tamer Başar, "Solutions to a Class of Linear-Quadratic Gaussian (LQG) Stochastic Team Problems with Nonclassical Information," *Systems Control Letters*, Vol. 9, No. 2, July 1987; pp. 125-130.

*[P18] Rajesh Bansal and Tamer Başar, "Simultaneous Design of Communication and Control Strategies for Stochastic Systems with Feedback," *Proceedings, 8th International Conference on Analysis and Optimization of Systems*, Antibes-Juan Les Pins, France, June 8-10, 1988.

*[P19] Tamer Başar, "Solutions to a Class of Nonstandard Stochastic Control Problems with Active Learning," *IEEE Trans. on Automatic Control*, to appear, 1988/89.

*[P20] Rajesh Bansal and Tamer Başar, "Joint Estimation and Control for a Class of Stochastic Dynamic Teams," submitted to the *1988 IEEE Conference on Decision and Control*, Austin, Texas, December 1988.

*[P21] Rajesh Bansal and Tamer Başar, "Joint Design of Communication Channels and Decision Policies in Stochastic Teams," presented at the *IEEE CSS Workshop on Decentralized and Distributed Control*, Columbus, Ohio, September 14-15, 1987; a fuller version of the paper is currently being written.

*[P22] Rajesh Bansal and Tamer Basar, "On Problems Exhibiting a Dual Role of Control," submitted to the *1988 IIEEE Conference on Decision and Control*, Austin, Texas, December 1988.

*[P23] Rajesh Bansal and Tamer Başar, "Communication Games with Partially Soft Power Constraints," *Journal of Optimization Theory and Applications*, Vol. 61, No. 3, June 1989.

*[P24] Rajesh Bansal and Tamer Basar, "Optimum Encoding and Decoding Policies for Jammed Communication Channels with Soft and Partially Soft Power Constraints," *Proceedings, 22nd Conference on Information Sciences and Systems*, Princeton University, Princeton, New Jersey, March 16-18, 1988.

*[P25] Tamer Basar and Rajesh Bansal, "The Theory of Teams: A Selective Annotated Bibliography," July 1987, submitted.

*[P26] Tamer Basar, "Some Thoughts on Rational Expectations Models, and Alternate Formulations," to appear in a special issue of *Computers and Mathematics with Applications*, 1988/89.

*[P27] Tamer Basar, "Dynamic Optimization of Some Forward-Looking Stochastic Models," to appear in the *Proceedings of Bellman Continuum*, to be published by Springer Verlag in November 1988.

Theses

[T1] Shu Li, "Recursive Schemes for Optimal Policies in Distributed Decision Making," M.S. Thesis, University of Illinois, Urbana-Champaign, July 1985.

[T2] Derya H. Cansever, "Incentive Control Strategies for Decision Problems with Parametric Uncertainties," Ph.D. Thesis, University of Illinois, Urbana-Champaign, August 1985.

[T3] Rajesh Bansal, "Solutions to some Stochastic Control Problems with Nonclassical Information," M.S. Thesis, University of Illinois, Urbana-Champaign, January 1986.

[T4] Rajesh Bansal, "Simultaneous Communication and Control Strategies in Stochastic Control and Team Problems," (tentative title) Ph.D. Thesis, University of Illinois, Urbana-Champaign, to be submitted in July 1988.

4. Conference Presentations and Plenary Talks (May 1, 1985 - April 30, 1988)

[C1] Paper [P5] was presented at the *1985 American Control Conference*, Boston, Massachusetts, June 1985.

[C2] Paper [P7] was presented as an opening talk at the *7th Annual Conference on Economic Dynamics and Control*, London, England, June 25-27, 1985.

[C3] Tamer Başar, "Decentralization and Incentives in Organizations," presented at the *1986 IFAC Workshop on Games and Decisions*, Beijing, China, August 1986, as a plenary talk.

[C4] Paper [P8] was presented at the *1986 IFAC Workshop on Games and Decisions*, Beijing, China, August 1986.

[C5] Paper [P5] was presented at *Optimization Days '86*, April 30-May 2, 1986, Montreal, Canada.

[C6] Derya H. Cansever and Tamer Başar, "Sensitivity Analysis in Multi-Stage Dynamic Games," presented at *Optimization Days '86*, April 30-May 2, 1986, Montreal, Canada.

[C7] Shu Li and Tamer Başar, "A Distributed Algorithm for the Computation of Nash Equilibria in Linear Stochastic Differential Games," was presented at the *1986 Conference on Information Sciences and Systems*, March 1986, Princeton, NJ, and the paper appeared in the Conference Proceedings, pp. 652-656.

[C8] Rajesh Bansal and Tamer Başar, "Solutions to Some Stochastic Team Problems and Zero-Sum Games with Nonclassical Information Arising in Communication Systems," presented at the *1986 International Information Theory Symposium*, Ann Arbor, Michigan, October 1986.

[C9] Paper [P9] was presented at the *1986 American Control Conference*, Seattle, Washington, June 1986.

[C10] A preliminary version of paper [P16] was presented at the *1986 American Control Conference*, Seattle, Washington, June 1986, and it appeared in the Conference Proceedings, pp. 45-50.

[C11] Paper [P15] was presented at the *25th IEEE Conference on Decision and Control*, Athens, Greece, December 1986; a shortened version appeared in the Conference Proceedings, pp. 243-247.

[C12] Tamer Basar, "Some Solvable Instances of NP Complete Control Problems with Nonclassical Information," invited paper at *2nd Annual Bilkent Symposium on Computer Science and Operations Research*, Ankara, Turkey, June 22-23, 1987.

[C13] Tamer Basar, "Asynchronous Algorithms in Noncooperative Games," *1987 Conference on Economic Dynamics and Control*, Newton, Massachusetts, June 24-26, 1987.

[C14] Paper [P19] was presented at the *Conference on Stochastic Games*, Chicago, Illinois, June 26-27, 1987.

[C15] Tamer Basar, "Stochastic Team Theory," plenary talk given at *Summer Institute on Dynamic Optimization*, Dublin, Ireland, July 6-17, 1987.

[C16] Paper [P11] was presented at the *10th IFAC World Congress*, Munich, West Germany, July 27-31, 1987.

[C17] Paper [P12] was presented at the *10th IFAC World Congress*, Munich, West Germany, July 27-31, 1987.

[C18] Paper [P21] was presented at the *IEEE CSS Workshop on Decentralized and Distributed Control*, Columbus, Ohio, September 14-15, 1987.

[C19] Paper [P14] was presented at the *1987 IEEE Conference on Decision and Control*, Los Angeles, California, December 9-11, 1987.

[C20] Tamer Basar, "Stochastic Control Problems with Nonclassical Information," presented as an invited paper at the *INDO-US Workshop on Signals and Systems*, Bangalore, India, January 8-12, 1988.

[C21] Paper [P24] was presented at the *22nd Conference on Information Sciences and Systems*, Princeton, New Jersey, March 16-18, 1988.

5. Attached Reprints and Preprints